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HEAT AND MASS TRANSFER BETWEEN GAS AND GRANULAR MATERIAL

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Аннотация—В статье рассмотрена задача тепло- и массообмена между твердыми частицами, лежащими в плотном слое, и газом, протекающим (фильтрующимся) через этот слой.

Из уравнений, определяющих перенос вещества в неизотермических условиях, разработан метод определения интенсивности процесса из экспериментальных данных по изменению концентрации реагирующего газа при гетерогенной реакции в слое и, в частности, в слое горящих коксовых частиц.

Получены из опытных данных по горению углерода характеристические функции тепло- и массообмена.

$$Nu = f(Re_f),\ \phi = f_1\ (Re_f)$$

Доказана их однозначность для переноса тепла и вещества. Разработана система расчета газообразования в реагирующем слое и теплообмена при нагреванииии или охлаждении твердых частиц. Показана высокая интенсивность процессов тепло- и массообмена между твердыми частицами в плотном слое и газом, движущемся через этот слой.

NOMENCLATURE

ϕ,	transfer function;
τ,	time;
x, y, z,	co-ordinates;
W_x, W_y, W_z,	components of gas velocity corresponding to the co-ordinates;
$W_{c.f.}$	cold filtration velocity;
c,	concentration of reacting component;
c_s,	gas concentration on particle surface;
D,	molecular diffusion coefficient;
$\Delta_1, \Delta_2 \ldots \Delta_n$,	chemical "sources" and "sinks";
T_o,	gas temperature;
T_b,	temperature of reaction surface;
F_c,	contour surface of particles;
K_r,	reduced constant of reaction rate;
β,	mass transfer coefficient;
α,	heat transfer coefficient;
a_b,	quantity characterizing pore volume of bed (proportional to porosity);
d,	particle diameter;
Z,	length of reaction zone.

Dimensionless numbers

$Nu_{\min}$	limited minimum Nusselt number;
Nu,	Nusselt number;
Re,	Reynolds number;
Sm,	Semenov number;
Pe,	Peclet number;
Pr,	Prandtl number;
Re_f,	cold filtration velocity Reynolds number.

1. HEAT AND MASS TRANSFER AT GAS MOTION IN A PACKED BED OF SOLID PARTICLES

THE PRACTICAL application of heat and mass transfer processes between fine solid particles and a gas, to laboratory and industrial processes increases from year to year [1]. In this case three prominent heat and mass transfer problems between a gas and particles are of the greatest interest:

1. In a packed bed of solid particles through which a gas flow passes.
2. In gas suspension, i.e. in a parallel flow of particles and a gas.
3. In a "fluidized" bed when a gas flow passes through a bed of particles upwards with velocities lower than the free-falling velocity of solid particles but higher than the velocity for which the particles in a packed bed remain motionless.

In order to design adequate effective industrial apparatuses it is necessary to work out a method of engineering calculation for heat and mass transfer processes between solid particles and a gas both in a gas suspension and in packed and fluidized beds.

It is known that heat and mass transfer is described by similar differential equations. Under the conditions of a reacting homogeneous system the change in gas concentration with time (τ) may be approximately written [2] as follows:

$$\frac{\partial c}{\partial \tau} = W_x \frac{\partial c}{\partial x} + W_y \frac{\partial c}{\partial y} + W_z \frac{\partial c}{\partial z} - D\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial z^2}\right) + \varDelta_1 c + \varDelta_2 c + \ldots \varDelta_n c. \tag{1}$$

For a directed one-dimensional flow, for example, along a cylindrical reactor and under stationary conditions, equation (1) may be written in the following simplified form:

$$\varDelta = W_z \cdot \frac{\mathrm{d}\bar{c}}{\mathrm{d}z} - D\frac{\mathrm{d}^2\bar{c}}{\mathrm{d}z^2}. \tag{2}$$

Equation (2) is also applicable with fairly high accuracy to the heterogeneous system with fine solid or liquid particles uniformly distributed in a gas.

It is necessary to bear in mind that from a strictly theoretical point of view concentration c with mass transfer, as well as temperature T with heat transfer, will not be exactly the same along the axes x and y, i.e. through the cross section normal to z, and the isothermal surfaces will inevitably be wavy, because of localized sources and sinks. However, the finer the solid (liquid) particles, the more uniform distribution of particles in a gas, and the higher their concentration in a gas, the more exact will be equality $c \approx \bar{c}$ with any given value of z, although prior to the end of a heterogeneous reaction, $\bar{c}$ cannot be, strictly speaking, equal to c nor can $\bar{T} = T$.

At first sight, it seems quite erroneous to consider the problem (for example, on mass transfer in a heterogeneous reaction) to be one-dimensional, since the reacting substance itself is transferred also from the gas to the solid (liquid) surface of particles, thus the system is certainly not one-dimensional. Nevertheless, in this case an analytic one-dimensional treatment of the process is not only approximately valid, but gives adequate results in practice. If we consider the reactions connected with mass transfer to a surface of particles as "source" and "sinks" and determining their laws, it becomes possible to solve the problem of determining the change in reactant concentration along length Z of a reaction zone. Such formulation of the problem allows the use of equation (2) for estimating the change in a gas concentration when reacting in a gas suspension, in packed and fluidized beds and even in a channel with walls which constitute a reaction surface [2].

For simplicity consider one heterogeneous reaction which is governed by the following kinetic equation

$$\varDelta = \beta \cdot \bar{c} \cdot F_c \cdot \frac{1}{1 + Sm}; \quad Sm = \frac{\beta}{K_r}; \tag{3}$$

where β is the mass transfer coefficient in equations

$$\frac{\mathrm{d}c}{\mathrm{d}\tau} = \beta(\bar{c} - c_s) \cdot F_c = K_r \cdot c_s F_c. \tag{4}$$

Since $(\bar{c} - c_s)$ is the concentration head, i.e. the difference of mass fractions of a reacting gas in the volume $\bar{c}$ and on the surface c_s the value β is the specific reaction rate (per unit F_c) with the concentration head equal to unity. Taking into account that, on the other hand, K_r is a specific reaction rate at c_s equal to unity, one can see that the dimensionless quantity Sm (the Semenov number) represents [2] the ratio of specific rates of the mass transfer process and of a heterogeneous chemical reaction.

Solving equation (3) with corresponding boundary conditions, we obtain for isothermal conditions and great excess of solid reagent, for example, in a packed bed of small particles blown by a gas†

$$c_Z = c_o \cdot \exp\left\{-a \cdot F_c \cdot Z \frac{1}{1 + Sm} [\sqrt{(Pe^2 + B \cdot Nu)} - Pe]\right\}. \quad (5)$$

When diffusion transfer along the flow is relatively small compared with that caused by forced gas motion, the solution of the equation will be of the following form:

$$c_Z = c_o \,.\, \exp\left[-\frac{Nu}{Pe} \cdot F_c \cdot Z \frac{1}{(1 + Sm)}\right] \quad (6)$$

and for a diffusion region at $Sm < 0{\cdot}1$

$$c_Z \approx c_o \cdot \exp\left(-\frac{Nu}{Pe} \cdot F_c \cdot Z\right) = c_o \exp\left(-\phi \cdot F_c \cdot Z\right). \quad (7)$$

The equations obtained are applicable to calculate concentration fields of the isothermal heterogeneous reaction of a gas with fine particles (granular material) both for packed and fluidized beds and for a gas suspension. With increase in size of solid particles the accuracy of calculation by the equations obtained decreases somewhat. However, since the transfer intensity under various conditions acquires a single-valued expression through Nu or the transfer function ϕ, the agreement of experimental data with the predicted values should be also quite satisfactory.‡ Whether the reaction is isothermal or no is of great importance in determining transfer intensity.

This effect in the diffusion region (at $c_s \approx 0$) is accounted for with accuracy sufficient for practice by the following equation [2] for the turbulent region:

$$c_Z = c_o \frac{T_o/T_b}{\exp(\phi \cdot F_c \cdot Z) - 1 + T_o/T_b}. \quad (8)$$

With small difference $(T_b - T_o)$ equation (8) is converted into equation (7), whilst with great non-uniformity of the temperature (when T_o/T_b is considerably smaller than unity) the difference in the process of concentration change of the reacting gas components obtained according to these equations becomes very large.

It is quite obvious that in the diffusion region the concentration distribution c over the height of the bed or the flow length of the gas suspension (one-dimensional problem) will in fact be largely determined only by the physical parameters and, in the first place, by the value of the transfer function ϕ. It follows that, ϕ (transfer intensity) can be actually determined, when the distribution of reacting gas concentrations is measured along the length of the reaction zone Z.

From the theoretical analysis of the mechanism of heat and mass transfer processes [3] and [2] it may be shown that for a gas (when Pr is close to unity) the function ϕ in the form

$$\phi \rightrightarrows f(Re) \text{ for } Pr \approx \frac{\nu}{D} \approx \frac{\nu}{a} \approx 1 \quad (9)$$

for particular hydrodynamic conditions (packed bed, gas suspension, fluidized bed, channel, separate particle) will be single-valued and equal for both heat and mass transfer.

This allows us to determine relation (9) which is necessary for calculations either in experiments on heat transfer or from the experiments on mass transfer depending on which is the simpler and is more amenable to measurement. However, before using equation (9) obtained from experiments on mass transfer and vice versa, we must verify the validity and accuracy of such calculations by comparing the results by direct experimental investigation of this function on heat and mass transfer, for example, for the case of a packed bed of granular material.

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† $Nu = \frac{\beta \cdot d}{D}$ (for diffusion, i.e. mass transfer);

$Nu = \frac{\alpha \cdot d}{\lambda}$ (for heat transfer);

$Pe = \frac{W \cdot d}{D}$ and $Pe = \frac{W \cdot d}{a}$ (respectively).

‡ Provided that values of Nu and ϕ are correctly determined.

2. HEAT AND MASS TRANSFER IN A PACKED BED OF SOLID PARTICLES

An experimental determination of heat transfer rate between solid particles in a bed and a gas flowing through this bed is a very complicated problem. Simultaneous change in the gas and particle temperature through the height of the bed and with time makes us use [4, 5], rather a complicated method of estimating heat transfer coefficients from experimental curves. This method of finding the value of α by this method is comparatively inaccurate, and scattering of experimental data is considerable both because of inaccuracy in calculation and difficulty inherent in testing.

Attempts to simplify the problem by maintaining the temperature of moist solid particles constant, whilst being dried, have led however, to the complication of the heat transfer process itself by the effect of phase change (evaporation) processes and by the motion of the evaporated substance from the surface of heated particles.

All this has led to considerable inaccuracy in determining the relation of *Nu* to *Re* which is illustrated by the data in Fig. 1 where results of various investigations on heat transfer in a packed bed of particles are given. From Fig. 1 one can see that scattering of experimental points is considerable.

It should be noted that the study of heat transfer between a gas and a bed of coarse particles (high values of $Re_f > 200$) is also complicated by the influence of the internal heat resistance of particles which have low heat conductivity.

Some serious difficulties when studying the heat transfer processes are eliminated when the process of mass transfer from a gas to the surface of solid particles is experimentally investigated.

When the velocity of a chemical reaction is high ($Sm < 0{\cdot}1$) in the diffusion region, where a heterogeneous reaction occurs, the concentration of the reacting gas near the surface (c_s) is close to zero [2] and it is only the value of the average concentration of the reacting gas ($\bar{c}$) in a gas flow which is therefore variable over the length of the reaction zone (in a bed). Usually it is not difficult to measure experimentally the distribution of mass fraction over the height of the reacting bed of solid particles $[\bar{c}_Z \Rightarrow f(Z)]$ under quite specific conditions which are under strict control.

Having experimental points or curves $[\bar{c}_Z \Rightarrow f(Z)]$ recorded by automatic instruments and using equation (8), it is not difficult to determine values of *Nu* and the transfer function ϕ. It is of course possible provided that equation (8) corresponds to the mechanism of the mass transfer process investigated.

From the experimental data, values of ϕ and *Nu* are determined in the following way.

For heat transfer in a packed bed the value of the contour surface of solid particles, F_e, in a cubic metre volume of the bed is defined by the following relation

$$F_e = \frac{a_b}{d} \tag{10}$$

where the value of a_b usually varies between 3·0–4·0 depending on the shape of particles and their arrangement in the bed. For the usual arrangement of irregularly shaped particles, for

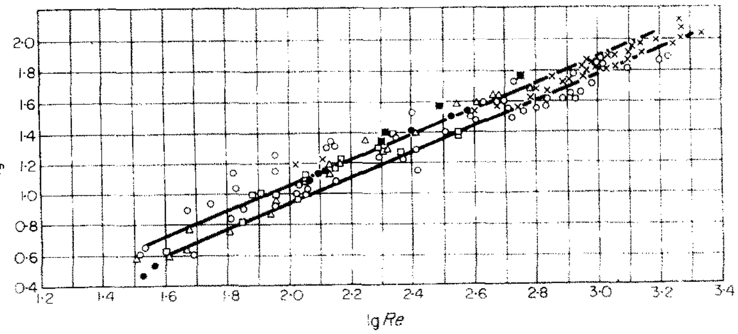


FIG. 1. The effect of Re_f on *Nu* according to the data of various authors.

example, of coal, the value of a_b is approximately 3·8 found experimentally [5], and, consequently, in this case designating:

$$T_o/T_b = T_b^o \tag{11}$$

$$c_Z/c_o = c_o^Z \tag{12}$$

and

$$3{\cdot}8\phi = x^* \tag{13}$$

equation (8) may be written in the following form convenient for calculations

$$c_o^Z = \frac{T_b^o}{e^{x^* Z/a} + T_b^o - 1}\,. \tag{14}$$

Using a computer, one may easily calculate and plot curves $c_o^Z \rightrightarrows f(Z/d)$ for various values of T_b^o and x^*, i.e. curves of relative concentration of reacting gas in a bed with respect to the dimensionless height of the packed bed of solid particles. In Figs. 2, 3 and 4 such predicted curves for three values of T_b^o equal to 0·15, 0·4, 1·0, respectively are given as an example.

Curves calculated at various values of x^*, i.e. at various values of the transfer function ϕ, are shown in each figure.

Comparison between Figs. 2, 3, and 4 clearly shows how great the significance of non-uniformity of a heterogeneous process is for the intensity of mass transfer in a turbulent boundary layer. For a degree of gas reaction, equal to 75·0 per cent, for example of the initial concentration at a height (Z/d) of the bed equal to two at $T_o/T_b = 1{\cdot}0$ (isothermal conditions) it is necessary that $x^* = 0{\cdot}7$ [$\phi = (0{\cdot}7/3{\cdot}8) = 0{\cdot}183$] whilst at $T_o/T_b = 0{\cdot}15$ this very degree of gas reaction is achieved at $x^* = 0{\cdot}18$ ($\phi = 0{\cdot}05$), i.e. when the value of the transfer function is approximately five times less. It should be noted that increase of non-uniformity of the reaction temperature, i.e. increase in the difference between the initial temperature of a reacting gas and that of a reaction surface strongly influences the shape of the curves $c_o^Z \rightarrow f(Z/d)$ themselves.

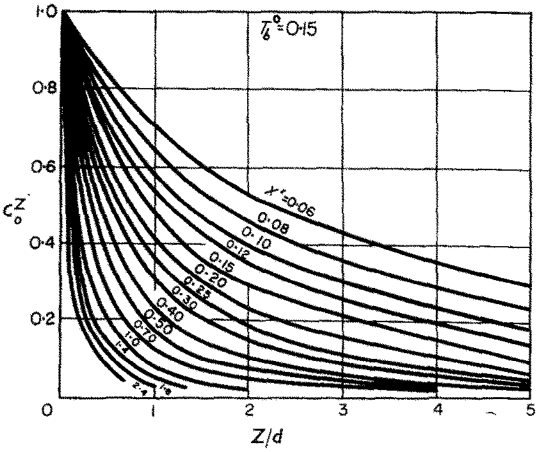


FIG. 2. Change in relative reacting gas concentration based on dimensionless bed height at $T_b{}^o = 0{\cdot}15$ and various x^*.

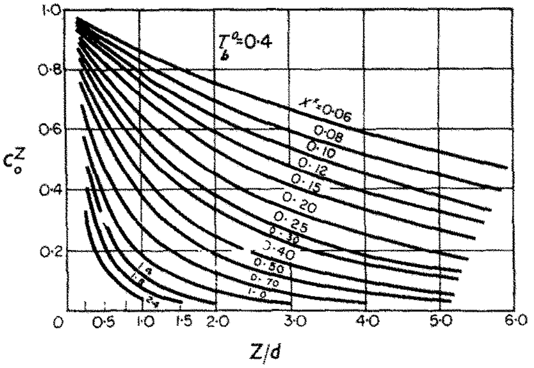


FIG. 3. Change in relative reacting gas concentration based on dimensionless height at $T_b{}^o = 0{\cdot}4$ and various x^*.

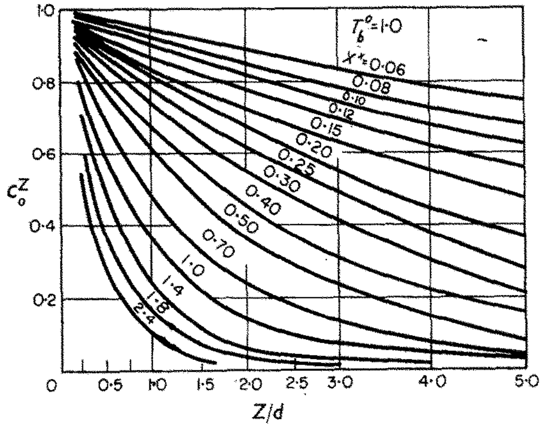


FIG. 4. Change in relative reacting gas concentration based on dimensionless bed height at $T_b{}^c = 1{\cdot}0$ (isothermal conditions) various x^*.

3. MASS TRANSFER WITH COMBUSTION OF A PACKED BED OF CARBON PARTICLES

Combustion of a carbon bed is a typical heterogeneous reaction where the peculiarities of transfer of oxygen and the reaction product when a gas flow moves in a packed bed of coke (carbon) particles are clearly revealed. Owing to highly exothermal nature of the combustion reaction, the influence of non-uniformity of the process temperature on mass transfer rate becomes apparent.

An experimental investigation of mass transfer rate in the combustion process is more convenient if conducted in the diffusion region when the value of the Semenov number (*Sm*) is close to zero, i.e. the chemical reaction rate is high and does not decrease the total rate of the reaction between carbon and oxygen. Investigation in the diffusion region is also of great importance, since in this case the surface temperature of solid particles in the oxygen combustion is actually the same over the whole height and from the point of view of non-uniformity of the reaction temperature, equation (8), therefore, closely reflects the real combustion conditions.

The author showed [2] that with carbon combustion in a bed of coke particles two sharply different regimes of oxygen consumption are possible:

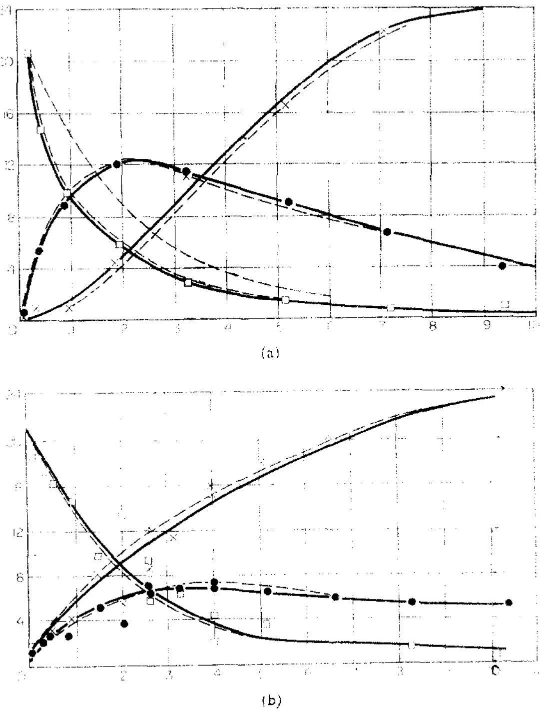
1. Purely heterogeneous combustion when volumetric reactions hardly take place and the whole quantity of oxygen is consumed on the carbon surface.
2. A homogeneous–heterogeneous combustion when a part of oxygen is taken up in the interaction with carbon oxide in the gas volume.

To exact investigations of the mass transfer process only the first regime is applicable, a process which takes place with coke particles no more than 15 mm in size under the ordinary conditions of combustion at $W_{c.f.}$ (cold filtration rate) up to 1·0–2·0 m/s and with small particles less than 7–10 mm in size at high blast velocities [2] more than 0·4–0·6 m/s and at low velocities less than 0·1–0·15 m/s. When coarse carbon particles and fine particles are burnt with low blast velocities oxygen is not consumed in the volume, since the whole carbon monoxide burns near the surface of carbon particles, and the mechanism of oxygen and carbon transfer processes corresponds closely to pure surface heterogeneous reaction.

At high blast velocities and with fine carbon particles the so-called high-rate of gasification in the oxygen region takes place in which the volumetric oxygen flow is also absent owing to the effect of the carbon monoxide carried away from the combustion region.

From Figs. 5 and 6 one can see that the effect of volumetric combustion of carbon monoxide and additional consumption of oxygen is quite clear.

Fig. 5 shows gas formation curves for two experiments carried out under the same conditions, the only difference between them being that in one of them (upper plot) carbon monoxide, forming with combustion, reacted with oxygen and in the other it was carried away from the oxygen region. Comparison of the figures and the curves of oxygen concentration



(a)

(b)

FIG. 5. Gas formation in a bed of burning coal with afterburning of oxides of carbon (upper) and without volume consumption of oxygen (lower).

change given in the upper plot, shows that when carbon monoxide is burnt up the oxygen consumption greatly increases and the oxygen region decreases.

Fig. 6 represents an experiment conducted by Karzhavina [2] with combustion of carbon pellets 10 mm in size where in the second half of the combustion zone carbon monoxide reacts with oxygen, and the law of oxygen consumption is modified, i.e. the intensity of oxygen combustion definitely increases.

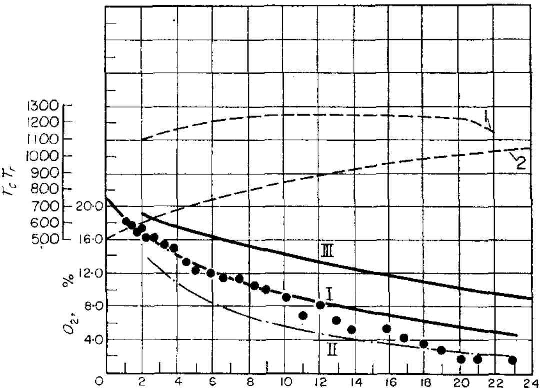


FIG. 6. Gas formation in a burning bed of charcoal pellets 10 mm in size.

From the present analysis and the examples mentioned above it is obvious that for obtaining exact and reliable data on the transfer intensity (ϕ, Nu) from experiments on combustion, it is necessary to ensure conditions of heterogeneous interaction between carbon and oxygen in the diffusion region as well as the constancy of exactly measured values T_o, T_b, $W_{c.f.}$ and Re_f, size particles, layer height and concentrations $\bar{c}_Z$ and c_o. Such data are quite realizable from experiments where quantities Re_f and Pe_f change over extremely wide ranges. Numerous investigations on combustion [6, 7], some of which may be used for determining ϕ and Nu, are reported.

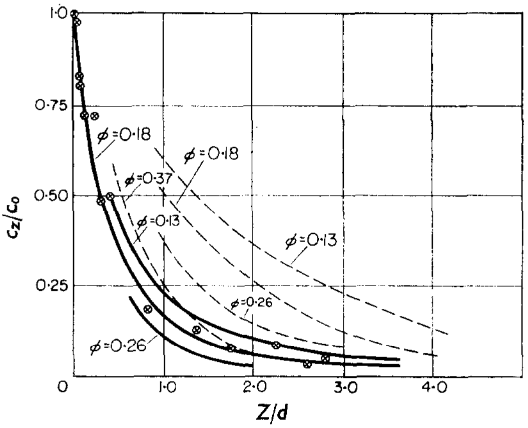


FIG. 7. Determination of the value of the transfer function from the experimental data $c_Z/c_o \xrightarrow{\sim} f(Z/d)$.

Fig. 7 gives experimental data obtained by Karzhavina from two experiments conducted under the same conditions at a vapour–air blast velocity $W_{c.f.} = 1{\cdot}0$ m/s and temperature $T_b = 1400$°C. Theoretical curves are plotted at various values of ϕ for two conditions $T_o/T_b = 1{\cdot}0$ and non-isothermal $T_o/T_b = 0{\cdot}2$, similar to the actual conditions of the experiments illustrated in Fig. 7.

Comparison between the theoretical curves and experimental points proves that the theoretical curves for the isothermal conditions (dashed lines) differ considerably from the experimental data, whereas the predicted curves for the non-isothermal conditions, corresponding to the experimental ones, agree well with the experimental data.

Fig. 7 shows how to determine ϕ comparing the experimental data with a group of predicted curves for various values of ϕ, three predicted curves for the values of $\phi = 0{\cdot}13$; $0{\cdot}18$ and $0{\cdot}26$ ($x^* = 0{\cdot}57$, $0{\cdot}7$ and $1{\cdot}0$ respectively) are presented in this figure.

Experimental points lie well between the curves, corresponding to $\phi = 0{\cdot}13$ and $\phi = 0{\cdot}18$ so that the experimental data more exactly satisfy $\phi = 0{\cdot}16$ and $Nu = \phi \,.\, Pe = \phi \,.\, Re_f \,.\, Pr = 9{\cdot}1$, respectively.

Thus, from the experiment on combustion of a packed bed of carbon particles at Re_f† = 78, the intensity of mass transfer from a gas to solid particles is determined by values of $Nu = 9{\cdot}1$ and the transfer function $\phi = 0{\cdot}16$. The values of ϕ and Nu may be determined by this method using nomograms, similar to those in Figs. 2, 3

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† Re_f is determined for the temperature T_b, i.e. for that of the carbon particles.

and 4, for all the experimental data, in which the distribution of oxygen concentration over the height of the burning carbon (coke) bed was measured. The more exact the experimental data and the experimental conditions, the more exact the mass transfer intensity, i.e. values of Nu may be determined.

Some results for determining Nu which were obtained by the method used for determining Re_f from unity to 260 are presented in Fig. 8. Here the experimental points are computed from the experimental data [8, 2] on combustion of coke particles from 2 to 30 mm in size for blast velocities (air enriched or depleted with oxygen and steam–air) $W_{\mathrm{c.f.}}$ from 0·02 to 1·4 m/s and at temperatures between 1000 and 1700°C.

In Fig. 8 the curve is plotted for $Re_f > 40$ from the equation

$$Nu = 0{\cdot}24\, Re^{0{\cdot}83}. \tag{15}$$

This equation describes satisfactorily (as it is shown in Fig. 1) the experimental data on heat transfer between a gas flow and a surface of solid particles in a packed bed through which the gas flows and filters. One can see from Fig. 8 that the experimental points obtained from the experiments on combustion lie well on the curve plotted according to equation (15).

The result obtained shows that in a gas flow moving in a packed bed the relation of Nu and Re_f has the same value both for heat transfer and mass transfer. This allows this function to be found experimentally, necessary for calculations from experiments on both heat and mass transfer.

The data obtained from the mass transfer experiments may be used not only for engineering but for more accurate computations of the processes of gas heating of particles in a packed bed, and vice versa. It can also be applied to calculations of the length of the reaction zone taking mass transfer into account.

When there exists heat and mass transfer between the gas and packed bed of solid particles, then Nu and ϕ may be determined with sufficient accuracy by equation (15) for Re_f from 40 to 260.

The conclusion on single-valuedness (for the turbulent region) of the functions $Nu \rightrightarrows f_1(Re)$ and $\phi \rightrightarrows f_2(Re)$ for heat and mass transfer in a packed bed is also borne out for the processes taking place in the so-called "fluidized" bed and in gas suspensions, i.e. in a parallel flow of a gas with solid or liquid particles.

Experimental data presented in Fig. 8 diverge considerably from the predicted values [2] at $Re_f <$ 40–30, for which the transition from the turbulent boundary layer (a turbulized region) to a laminar jet flow is observed in a packed bed of solid particles according to the data obtained by many investigators.

When the value of Re_f decreases, the effect of the molar heat and mass transfer process from a gas to a surface of particles continuously decreases, and starting from $Re_f \approx 40$ (for a packed bed) the transfer intensity is determined by pure-molecular heat conduction and diffusion processes.

It is well-known that for heat and mass transfer

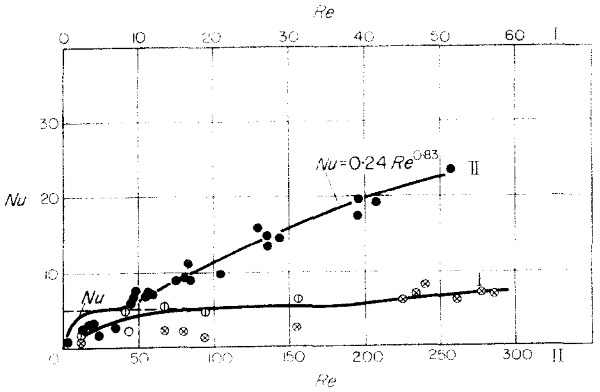

FIG. 8. The effect of Re_f on Nu according to the experimental data on combustion of a bed composed of carbon particles.

from a gas to a single particle in an infinite gas volume $Nu = 2$. However, in a packed or fluidized bed the boundary conditions of transfer processes are distinguished by the marked mutual influence of adjacent particles. In a packed bed when longitudinal transfer (over Z) is not taken into account, the limiting value of Nu is also specific, however this specific value is more than two.

If we assume the simplest way for determining $Nu_{\min}$ in a packed bed by considering every solid particle as a sphere bounded by an iso-concentration surface [2] determined by the value of porosity (free volume between particles), then for an infinite volume instead of $Nu = 2$ we obtain $Nu \approx 5$ for molecular transfer in a packed bed when solving the corresponding equation.

It has already been mentioned that with one-dimensional motion of a gas flow in a bed, a concentration (temperature) gradient arises along the flow, and heat and mass transfer along the flow should be taken into account, when determining the distribution of the concentrations and temperatures. For this purpose it is necessary to use a somewhat more involved equation (5) where we should have for $Nu_{\min}$

$$Nu_{\min} \approx 5 \qquad (16)$$

instead of equation (6).

In Fig. 8 the predicted curve for $Re_f < 40$ is plotted on the basis of the data of such calculation from the following equations taking into account longitudinal diffusion.

$$\phi = \frac{Nu}{Pe_f}; \quad \phi = \frac{1}{7{\cdot}2}[\sqrt{(Pe_f^2 - 15{\cdot}2\, Nu_{\min})} - Pe_f]. \qquad (17)$$

The broken line represents equation (16) which does not account for longitudinal diffusion.

We have already mentioned that the experimental values of Nu and ϕ for $Re_f < 50$, calculated from our experimental data using equation (8) for non-isothermal conditions, considerably from the predicted curve in equations (16) and (17). This divergence is quite natural, since the influence of non-uniformity of the process temperature on mass transfer by its nature differs greatly [2] for laminar and for turbulent regions of a gas flow.

For the turbulent region, as we have shown, non-uniformity of the temperature sharply intensifies mass transfer and changes the shape of curves $c_Z \rightarrow f(Z)$ while for the laminar region [2] the overall effect of non-uniformity of the process temperature is considerably weaker and owing to the much greater effect of Re_f on ϕ, it may decelerate the mass transfer process under certain conditions.

Besides the points obtained from equation (8), Fig. 8 presents the experimental points of Nu for small Re_f determined from equation (7), i.e. the influence of non-uniformity of the temperature on the mass transfer process is not taken into account. As is seen from comparison of these points with the predicted curve, the latter correlates with the experimental data quite satisfactorily.

The important practical conclusion on the high intensity in the heterogeneous process of heat and mass transfer under these conditions arises from our analysis of heat and mass transfer processes in a packed bed. It allows us to evolve high-intensity heat exchangers and chemical reactors.

When calculating heat and mass transfer processes, equation (15) for $Re_f > 40$ and equation (16) and (17) for $Re_f < 40$ should be used for determining Nu in the packed bed of solid particles.

In order to calculate the distribution of reacting gas concentration in reactors or temperature along the bed height of a heat

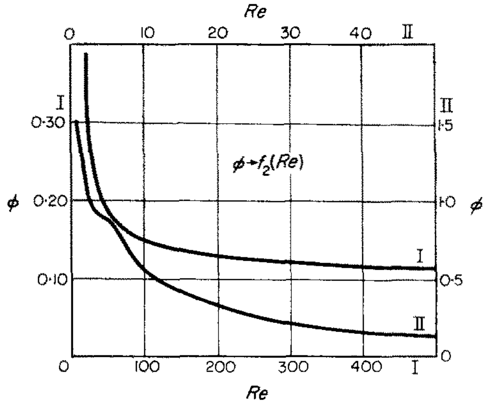

FIG. 9. The effect of Re_f on the transfer function ϕ for heat and mass transfer processes between gas and particles in a packed bed.

exchanger, ϕ should be determined from the plot in Fig. 9.

The gas formation process in a reacting bed of solid particles in the diffusion region ($Sm < 0{\cdot}1$) was calculated from equation (8).

At $Sm > 0{\cdot}1$ the "chemical deceleration" of the process should be also taken into account in accordance with equation (6) when calculating the gas formation.

In computation of the transfer process at $Re_f > 40$ non-uniformity of the process temperature of reaction should be always taken into account, and at $Re_f < 40$ it should not, i.e. the process is assumed to be isothermal, although it may not be isothermal in reality.

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Abstract—The paper deals with a problem of heat and mass transfer between the solid particles in a packed bed and the gas passing through this bed.

A method for determining the intensity of the process from experimental data on concentration change of a reacting gas in a heterogeneous reaction in a bed and, particularly, in a bed of burning coke particles, is worked out from equations governing mass transfer under non-isothermal conditions.

Characteristic values of heat and mass transfer are obtained on the basis of experimental data on carbon combustion:

$$Nu = f(Re_f),\ \phi = f_1(Re_f).$$

They are proved to be single-valued. A method for calculating gas formation in a reacting bed and heat transfer in the process of cooling or heating solid particles is developed. High intensity of heat and mass transfer processes between solid particles in a packed bed and a gas passing through this bed is shown.

Résumé—Cet article concerne le problème du transport de masse et de chaleur dans un lit de particules traversé par un courant de gaz.

On établit une méthode permettant la détermination de l'intensité des échanges, à partir des mesures de la variation de la concentration d'un gaz réactif, au cours d'une réaction hétérogène dans le lit, particulièrement dans un lit de particules de coke en combustion; cette méthode utilise les équations qui régissent le transport de masse dans des conditions non-isothermes.

Les valeurs caractéristiques du transport de masse et de chaleur sont obtenues à partir des données expérimentales sur la combusion du carbone

$$Nu = f(Re_f),\quad \phi = f_1(Re_f).$$

On montre qu'elles n'ont qu'une seule valeur. On expose une méthode de calcul de la formation de gaz dans un lit en réaction et des échanges thermiques au cours du chauffage ou du refroidissement des particules solides. L'intensité élevée des échanges dans un lit de particules traversé par un gaz est mise en évidence.

Zusammenfassung—Die Arbeit behandelt den Wärme- und Stoffübergang zwischen den Festkörpern im Festbett und dem durch dieses Bett strömende Gas.

Eine Methode zur Intensitätsbestimmung des Prozesses aus experimentellen Daten der Konzentrationsänderung des reagierenden Gases bei einer heterogenen Reaktion im Festbett, speziell in einem Bett aus brennenden Koksteilchen, liess sich aus Gleichungen für den Stoffübergang bei nicht isothermen Verhältnissen gewinnen.

Charakteristische Funktionen für Wärme- und Stoffübergang wurden auf Grund von Versuchsergebnissen über Kohlenstoffverbrennung gefunden:

$$Nu = f(Re_f); \quad \phi = f_1(Re_f)$$

Ihre Übertragbarkeit wird nachgewiesen. Eine Methode zur Berechnung der Gasbildung im Reaktionsbett und des Wärmeübergangs beim Abkühlen und Erwärmen der Festkörper ist angegeben. Die grosse Intensität der Wärme- und Stoffübergangsprozesse zwischen den Festkörpern des Festbetts und dem durchströmenden Gas wird gezeigt.